# Analysis of the overhead of <br> B.A.T.M.A.N. routing protocol in regular torus topologies 

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## Introduction

The overhead of a routing protocol is an important aspect, especially in wireless networks, where the medium is completely shared and the network capacity is limited [3]. This work concerns an analysis of the scalability of B.A.T.M.A.N., in terms of overhead, introduced by the OGM flooding mechanism.
The analysis has been done on regular torus topologies, providing also a generalization for the N-dimensional torus. Ring and Manhattan networks, have been analysed both theoretically and with simulation. The packet aggregation mechanism, included in the last versions, has been also tested with simulation, showing that this feature effectively reduce the overhead in dense networks.

## OGM overhead analysis

The OGM flooding mechanism used by B.A.T.M.A.N. seems to be not very scalable. In fact, in a network composed of $N$ nodes, the number of different messages flooding the network is exactly $N$ because every node sends out, at regular intervals, its own originator message. Each message is then rebroadcast by its neighbours and so on.
In a situation with no message losses, the number of OGM packet sent every OGM interval is $N^{2}$, because every node sends its OGM and forwards OGMs of the other $N-1$ nodes. It's clear that in a situation like this the protocol is not scalable at all.

| SYMBOL | MEANING |
| :--- | :--- |
| $d$ | number of dimension |
| $n$ | graph size parameter |
| $N$ | total number of nodes |
| $\mathcal{N}$ | set of all nodes |
| $E$ | total number of edges |
| $q$ | loss probability (uniform) |
| $p=(1-q)$ | successful transmission probability |
| $r_{D}(s)$ | number of nodes at distance $s$ in dimen- |
| $B$ | sion $D$ <br> $B$ |
|  | number of OGM broadcast at each node |
|  |  |

Table 1: Table of symbols used in the document.

In a wireless situation however, packet loss are rather frequent and not all the OGM packet will be received, so the expected number of OGM sent will be less that $N^{2}$.

In the next section we describe the notation, the network topology, and some other assumptions. Then the theoretical overhead is computed and compared with real measurements.

## Assumption and notation

The network topologies that we will analyse are regular torus networks in which the size in each dimension, that we call graph size parameter is equal. Denoting as $n$ the graph size parameter, the number of nodes in a regular torus of dimension $D$ is $N=n^{D}$, and the number of edges is $E=D N^{D}$.

Throughout the document a series of assumption that simplify the analysis will be used:

- Uniform loss probability: all link have the same loss probability $q$, the probability of successful transmission is $p=(1-q)$;
- Uniform OGM interval: all nodes have the same OGM interval;
- Shortest path stable routes: all nodes have a single shortest path stable route toward every other node, route changes are not considered.

The notation used in the document is summarized in table 1.

## Generalities

We can start calculating the expected number of messages sent out by a node in a generic network. First we have to consider that every node sends out its own OGM with probability 1 . The rebroadcast probability instead is the probability of having received the OGM from the best rout $母^{\text {1 }}$

The probability of receiving an OGM from a node via the best path is proportional to the path length. We express as $l_{o, k}$ the length of the optimal path from node $o$ to node $k$.
Indicating with $\mathcal{N}$ the set of node in the network, the average number of message broadcast for a single node is:

$$
B=1+\sum_{k \in \mathcal{N}}^{\mathcal{N} \backslash\{o\}} p^{l_{o, k}}
$$

With a constant failure probability, the protocol will results to be quite scalable thanks to the probability of receiving an OGM that decrease exponentially. The drawback is that it is improbable to receive OGM from far neighbour, with the consequence of not being aware of existence of distant nodes. For the simulations a probability dependant on the network dimension will be used.

## Bidirectional Ring network



Figure 1: A ring of dimension 12
Ring network is probably the simplest topology to imagine, it can be seen as an unidimensional torus. A ring with graph size parameter $n$, has $N=n$ nodes and $E=n$ links.

The distance distribution is very simple, we have that the number $r$ of nodes

[^0]at distance $s$ are:
\[

$$
\begin{array}{lll}
\text { n even: } & r=2 & s \leq \frac{n}{2}-1 \\
& r=1 & s=\frac{n}{2} \\
\text { n odd: } & r=2 & s \leq \frac{n-1}{2}
\end{array}
$$
\]

producing a number of broadcast $B$ in each node of:
n even: $\quad B=1+\sum_{s=1}^{\frac{n}{2}-1}\left(2 p^{s}\right)+p^{\frac{n}{2}}$
n odd: $\quad B=1+\sum_{s=1}^{\frac{n-1}{2}}\left(2 p^{s}\right)$
for which the summation formulas are:
n even: $\quad B=1+2 \frac{p\left(p^{\frac{n}{2}-1}-1\right)}{p-1}+p^{\frac{n}{2}}$
n odd: $B=1+2 \frac{p\left(p^{\frac{n-1}{2}}-1\right)}{p-1}$
and simplifying we obtain:

$$
\begin{array}{ll}
\text { n even: } & B=\frac{\left(p^{\frac{n}{2}}-1\right)(p+1)}{p-1} \\
\text { n odd: } & B=\frac{2 p^{\frac{n+1}{2}}-p-1}{p-1}
\end{array}
$$

## Bidirectional Manhattan network

Manhattan networks are a simple case of Torus Networks with only two dimension. Manhattan Networks have been widely studied in the past, especially with regards to find out optimal routing strategies. In the literature two types of Manhattan network have been studied, unidirectional and bidirectional. Our interest is focused on the bidirectional case that well fit the case of a number of node with omnidirectional antennas.


Figure 2: A $4 \times 4$ Bidirectional Manhattan Network

In a network having graph size parameter $n$, the total number of nodes is $N=n^{2}$ and total number of edges is $E=2 n^{2}$. Now we have to find out a distribution of shortest path distance in the network. Following [1], with $n>3$, the cardinality (indicated with $r$ ) of reachable set of nodes at distance $s$ from each node is:

$$
\begin{array}{lll}
\text { n even: } & r=4 s & s \leq \frac{n}{2}-1 \\
& r=4 s-2 & s=\frac{n}{2} \\
& r=4(n-s) & \frac{n}{2}+1 \leq s \leq n-1 \\
& r=1 & s=n \\
\text { n odd: } & r=4 s & s \leq \frac{n-1}{2} \\
& r=4(n-s) & \frac{n+1}{2} \leq s \leq n-1
\end{array}
$$

So the number of broadcast of a node can be expressed as:
n even: $\quad B=1+\sum_{s=1}^{\frac{n}{2}-1}\left(4 s p^{s}\right)+(2 n-2) p^{\frac{n}{2}}+\sum_{s=\frac{n}{2}+1}^{n-1}\left(4(n-s) p^{s}\right)+p^{n}$
n odd: $\quad B=1+\sum_{s=1}^{\frac{n-1}{2}}\left(4 s p^{s}\right)+\sum_{s=\frac{n+1}{2}}^{n-1}\left(4(n-s) p^{s}\right)$
Simplifying and applying summation formula for geometric and arithmetic-geometric
progression, a simpler expression can be obtained:
n even: $\quad B=\left(1-p^{\frac{n}{2}}\right)^{2}\left[\frac{(1+p)^{2}}{(1-p)^{2}}\right]$
n odd: $B=\frac{(1+p)^{2}}{(1-p)^{2}}+\frac{4 p^{\frac{n+1}{2}}\left(p^{\frac{n+1}{2}}-p-1\right)}{(1-p)^{2}}$

## Bidirectional three dimensional torus



Figure 3: A 3-dimensional torus of dimension 2

Analysis of the three dimensional case of Torus Networks is out of the scope of this work. Anyhow for completeness, a brief exposition of the distance distribution in such a network is reported.

Following the same procedure used for Manhattan networks, indicating the cardinality of reachable set of nodes at different distances with $r$, the number of nodes at shortest-path distance $s$ in a 3-dimensional torus network having graph size parameter $n$ (so $N=n^{3}$ nodes and $E=3 n^{3}$ edges), with $n \geq 3$ are:

$$
\text { n odd: } \begin{aligned}
r & =4 s^{2}+2 \\
& r=(3 n-2 s)^{2}-12(n-s)^{2}-1 \\
r & =(3 n-2 s)^{2}-1
\end{aligned}
$$

$$
s \leq \frac{n-1}{2}
$$

$$
\begin{aligned}
& \frac{n+1}{2} \leq s \leq n-1 \\
& n \leq s \leq \frac{3}{2}(n-1)
\end{aligned}
$$

$$
\begin{array}{rlrl}
\text { n even: } & r=4 s^{2}+2 & & s<\frac{n}{2} \\
r & =4 s^{2}-1 & s=\frac{n}{2} \\
r & =(3 n-2 s)^{2}-12(n-s)^{2}-4 & \frac{n}{2}<s<n \\
r & =(3 n-2 s)^{2}-1 & & s=n \\
r & =(3 n-2 s)^{2}+2 & & n<s<\frac{3}{2} n \\
r & =1 & & s=\frac{3}{2} n
\end{array}
$$

## Bidirectional N-dimensional torus

A generalization to the N -dimensional case can be done. It is sufficient to use the results described for a number of dimension less than 3 and use them to formalize and solve the problem recursively.
Denoting with $D$ the number of dimension, a torus of graph size parameter $n$ have a total number of nodes $N=n^{D}$ and a number of edges $E=D n^{D}$. Defining $r_{D}(s)$ as the number of nodes at distance $s$ in a torus with dimension $D$, we have that:

$$
\begin{aligned}
& r_{D}(0)=1 \\
& \forall D \\
& r_{1}(s)=2 \\
& s \leq\left\lfloor\frac{n-1}{2}\right\rfloor \\
& r_{1}(s)=1 \\
& \left\lfloor\frac{n-1}{2}\right\rfloor<s \leq\left\lfloor\frac{n}{2}\right\rfloor
\end{aligned}
$$

and recursively for $n$ odd:

$$
r_{D}(s)=r_{D-1}(s)+\sum_{i=1}^{\frac{n-1}{2}} r_{D-1}(s-i)
$$

and for $n$ even:

$$
r_{D}(s)=r_{D-1}(s)+\sum_{i=1}^{\frac{n}{2}-1} r_{D-1}(s-i)+r_{D-1}\left(\frac{n}{2}\right)
$$

This result comes from the observation that a torus of dimension $D$ and graph size parameter $n$, is equivalent to having $n$ torus of dimension $D-1$ that can be
"accessed" moving along one of the $D$ dimensions.
The number of broadcast at each node $B$ can be expressed as:

$$
B=\sum_{i=0}^{D\left\lfloor\frac{n}{2}\right\rfloor} r_{D}(i) p^{i}
$$

## Performance Measures

The measures have been done using a number of $q e m u$ virtual machines running an unmodified linux based operating system (OpenWrt) with B.A.T.M.A.N. (version 2010.1.0 compatibility version 11) integrated. The network links was simulated using VDE switch to connect various qemu instances, together with wirefilter to introduce packet loss probability. These network simulation tools are part of the VDE (Virtual Distributed Ethernet) project ${ }^{2}$.


Figure 4: Number of OGM messages sent out by every node every OGM_INTERVAL in a ring topology. Are reported the expected values and the measured ones both with aggregation enabled and disabled.

This configuration work perfectly for analysing protocol functioning and testing overhead, but is perfectly useless for testing bandwidth because all link are fullduplex, in other words every node can receive and transmit simultaneously which

[^1]is impossible in a real wireless network. Furthermore there is no way to simulate interference between neighbour transmissions.

The tests have been performed twice for every dimension of network, the first with the packet aggregation feature enabled(which is the default setting), and the second disabling it. The OGM_INTERVAL is set to 1 second while the other parameters has been set to the default values. Each test last 10 minutes.

The probability of successful transmission used in tests depends on graph size parameter $n$ :

$$
q=1-\frac{1}{n^{2}}
$$

this is for simulate a case in which the number of nodes increase proportionally to the node density of the network.

In figure 4 is shown the results of simulations on ring topology while in figure 5 are reported the results of simulations on Manhattan topology. Can be observed a linear relation between the network diameter and the number of OGM sent per OGM_INTERVAL.


Figure 5: Number of OGM messages sent out by every node every OGMINTERVAL in a Manhattan topology. Are reported the expected values and the measured ones both with aggregation enabled and disabled.

## Conclusion

From the results, it emerges that the proposed model fits well, the measured values are only slightly greater than the calculated ones. Another interesting observation is that the aggregation feature, included in the last B.A.T.M.A.N. releases, works well in reducing the number of OGM sent out by every node, so there is not a concrete room of improvement in these area. An interesting future work can be the analysis of the impact to the protocol overhead given by the presence of clients connected to B.A.T.M.A.N. nodes. The list of the connected nodes (HNA) in fact is written at the end of each OGM packet sent.

## References

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[2] Banerjee D., Mukherjee B., Ramamurthy S., "The multidimensional torus: analysis of average hop distance and application as a multihop lightwave network', Communications, 1994. ICC '94, SUPERCOMM/ICC '94, Conference Record, 'Serving Humanity Through Communications.' IEEE International Conference on, pp.1675-1680 vol.3, 1-5 May 1994
[3] Y. Yang, J. Wang, and R. Kravets, "Designing routing metrics for mesh networks", In WiMesh, 2005.
[4] Abolhasan M., Hagelstein B., Wang J.C.-P., "Real-world performance of current proactive multi-hop mesh protocols", Communications, 2009. APCC 2009, pp.44-47, 8-10 Oct. 2009


[^0]:    ${ }^{1}$ Remember that B.A.T.M.A.N. rebroadcast only OGM coming from the best route toward the originator. OGM received from secondary route are discarded.

[^1]:    ${ }^{2}$ http://vde.sourceforge.net/

